UNITED STATES BUREAU OF EDUCATION
BULLETIN, 1912, NO. 4 WHOLE NUMBER 472

MATHEMATICS IN THE TECHNICAL SECONDARY SCHOOLS IN THE UNITED STATES

INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

THE AMERICAN REPORT

COMMITTEE No. VI

WASHINGTON
GOVERNMENT PRINTING OFFICE
1912

INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS.

GENERAL OFFICERS.

PRESIDENT, F. KLEIN, GEH. REG.-RAT, THE UNIVERSITY OF GÖTTINGEN.
VICE PRESIDENT, SIR GEORGE GREENHILL, F. R. S., LONDON.
GENERAL SECRETARY, H. FEHR, THE UNIVERSITY OF GENEVA.

AMERICAN COMMISSIONERS.

DAVID EUGENE SMITH, CHAIRMAN, TEACHERS COLLEGE, COLUMBIA UNIVERSITY, NEW YORK, N. Y.
W. F. OSGOOD, HARVARD UNIVERSITY, CAMBRIDGE, MASS.
J. W. A. YOUNG, THE UNIVERSITY OF CHICAGO, CHICAGO, ILL.

THE AMERICAN REPORT.

Committee No. VI. Mathematics in the Technical Secondary Schools in the United States.

Professor C. N. HASKINS, Dartmouth College, Hanover, N. H., Chairman.
Professor F. D. CRAWSHAW, the University of Wisconsin, Madison, Wis.
J. E. DOWNEY, High School of Commerce, Boston, Mass.
Professor J. E. OSTRANDER, Massachusetts Agricultural College, Amherst, Mass.

Subcommittee 1. Public, Private, and Corporation Trade Schools.

Professor F. D. CRAWSHAW, the University of Wisconsin, Madison, Wis., Chairman.
Professor N. C. RIGGS, Carnegie Technical Institute, Pittsburgh, Pa.
Dean H. V. GUMMERE, Drexel Institute, Philadelphia, Pa.
Professor C. A. BENNETT, Bradley Polytechnic Institute, Peoria, Ill.
Principal C. F. BERRY, Milwaukee Trade School, Milwaukee, Wis.
E. C. BARKER, Polytechnic High School, Los Angeles, Cal.
E. R. SMITH, Brooklyn Polytechnic Preparatory School, Brooklyn, N. Y.

[The report of this subcommittee is incorporated in the general report.]

Subcommittee 2. Public and Private Commercial Schools.

J. E. DOWNEY, High School of Commerce, Worthington Street, Boston, Mass., Chairman.
Dr. J. T. ROREB, William Penn High School, Philadelphia, Pa.
RAYMOND G. LAIRD, Jamaica Plain, Mass.
Miss CLARA F. EATON, New York, N. Y.
RALPH D. BEETLE, Dartmouth College, Hanover, N. H.

Subcommittee 3. Agricultural Schools.

Professor J. E. OSTRANDER, Massachusetts Agricultural College, Amherst, Mass., Chairman.
Dr. D. L. STEVENS, West Raleigh, N. C.
Dr. TAIT BUTLER, Farm Gazette, Starkville, Miss.

Supplementary Report. The Industrial School of Secondary and Intermediate Grade.

NATHAN N. DICKLER, Manual Training High School, Brooklyn, N. Y.

CONTENTS.

PART I.

PART II.

MATHEMATICS IN THE TECHNICAL SECONDARY SCHOOLS IN THE UNITED STATES.

PART I.

INTRODUCTORY.

The secondary technical schools of the United States, because of their heterogeneity, present peculiar difficulties to an investigation along the lines laid down by the International Commission. While such schools have existed for many years, it is particularly within the last decade that a great increase in their numbers has taken place, for it is within that period that the tendency to break away from the traditions of the general secondary schools and to bring the schools into close contact with industrial and commercial life, rather than to raise to a maximum their efficiency in furnishing preparation for higher education, has become a movement of sufficient strength to alter essentially the character of existing schools and to determine that of those newly established.

The "manual training high school" is the oldest of the important types of public secondary technical schools in the country. As a type, moreover, it is the most conservative of the schools to be considered in this report, in that to a large extent the traditions of the general secondary school have been retained and the function of the school as an instrument of preparation for higher education emphasized. The ideals of this type of school are essentially scientific rather than classical or technical.

At the other extreme stands the "trades school," a type which is in its infancy as a public institution but examples of which have existed for many years as private or endowed institutions. Here the aim is primarily that of training for immediate entrance into a definite trade or trades and for efficiency in the work thereof. The school is not, or is not primarily at least, an intermediate step in the student's progress toward higher education.

These types represent the extremes. It must, however, be remembered that the lines of demarcation are by no means sharply drawn and that rigid classification is, at the present time at least, scarcely possible.

The schools just mentioned are in the province of subcommittee 1. The schools considered by subcommittee 2 fall into three classes—high schools of commerce, commercial departments of general secondary schools, and private commercial schools (the so-called "business colleges"). On these schools, too, though to a less extent, the influence of the traditions of the general secondary school is in evidence at one end of the series, while at the other end the ideal of the trades school, i. e., training for immediate business activity, is dominant.

The secondary agricultural schools studied by subcommittee 3 are of recent origin. More than schools of the other two classes they are supported in whole or in part by State rather than by municipal appropriations, and consequently are to a greater extent under State supervision. Their object is to provide such an education for the youth of the agricultural community as will tend to retain him in that community as an efficient member thereof. In consequence, except in so far as they lead to the agricultural colleges, their tendency is toward producing immediate vocational efficiency rather than to serve as a step toward higher education.

In view of this diversity it may well be asked what there is in common among these schools that justifies their inclusion in the same category. The obvious answer is that the schools in question are the most recent result of that movement which has led to the establishment of the technical colleges and the broadening of the curricula of the classical colleges and secondary schools, namely, the movement toward bringing the instruction within the school into closer contact with the phenomena and problems of life outside the school, and toward making the knowledge gained in the school more immediately useful to the pupil when he leaves.

AIM OF THE MATHEMATICAL INSTRUCTION.

With the wide differences in general object which exist in the technical secondary schools are naturally associated similar differences in the aim of the mathematical instruction therein.

The schools show in different degrees the common tendency to emphasize the utilitarian side of the subject. Large, well-organized schools which form an integral part of a municipal school system or are controlled by university authorities, while recognizing the utilitarian side and providing for it by suitable selection of problems and correlation with the work of the technical departments, also emphasize the logical element of the subject and the importance of mathematics as an independent science.

There are schools in which, if the character of the text used is any indication, the formal element is predominant; and on the other hand, as in the trades schools and the private commercial schools, the aim is mainly to produce accuracy and speed in the application of a limited range of mathematical principles to the problems of a definite trade or occupation.

The situation may be illustrated by the following quotations from information furnished by various schools:

(1) The ends to be attained are the knowledge of a body of geometrical truths to be used in the discovery of new truths, the power to draw correct inferences from given premises, the power to use algebraic processes as a means of finding results in practical problems, and the awakening of interest in the science of mathematics.

(2) In mathematics two ends are constantly kept in view: First, stimulation of the inventive faculty, exercise of judgment, development of logical reasoning, and the habit of concise statement; second, the association of the branches of pure mathematics with each other and with applied science, that the pupil may see clearly the true relations of principles and things.

(3) It is the aim * * * to give that knowledge and training to the students that shall make them capable men, ready to meet successfully the practical questions of everyday life, and to solve intelligently the problems constantly arising in office, factory, and field; hence, the practical side of mathematics is emphasized rather than the purely theoretical. Abstract mathematical discussions, as such, are avoided except as they are necessary to a better comprehension of results, and then they are made as direct and clear as possible.

Stress is placed upon the application to mechanical, physical, and electrical problems, but it is intended that the instruction shall be of such a character as to give the student power and incentive to perform ordinary mathematical work with confidence, precision, and success.

(4) In the courses in mathematics the main purpose is to train the students, not to prove propositions and formulæ, but to make intelligent use of these propositions and formulæ in the solution of original problems.

(5) They must know enough of mathematics, drawing, and science to insure intelligent, progressive workmanship, as contrasted with rule of thumb methods.

(6) We aim to give them some idea of the subject of elementary mathematics with special reference to its application in technical studies.

(7) The aim of the courses is twofold: First, to teach the methods of computation necessary for the solution of common problems arising in shop practice; second, to present in condensed form the essentials of algebra, geometry, and trigonometry for the benefit of those who have not had a high-school training, and to show the applications of these subjects to the more advanced types of shop problems.

(8) The practical results of the method are usually the acquisition of certain "rules of thumb" which are immediately available in the trade work of the student. There is probably no very great increase in mathematical ability.

(9) The course in mathematics is designed:

First, to develop in the pupil the power of independent thought, to cultivate the inventive faculty, and to inculcate the habit of clear, concise, logical statement. To this end the course is so arranged that the graphic, concrete branches of the subject precede those that are abstract and analytic.

Second, to teach the student the importance of mathematics in relation to the applied sciences, the mechanic arts, and to business life. For this purpose he is required to apply the formulas of algebra and trigonometry to physics, mechanics, chemistry, and engineering; and the short methods of arithmetic and mensuration to the practical work of bookkeeping and architecture.

The course in mathematics as taught in this school is both preparatory and complete. Those boys who finish their studies here possess a good working knowledge of the subject; and those who continue their studies in colleges or in technical schools possess an adequate preparation for higher work.

(10) The aim of the instruction is to inculcate habits of accuracy, rapidity, and neatness in the manipulation of algebraic operations, and to inspire a thorough knowledge of the fundamental principles and laws of the subject. To aid in securing these results the pupils are required to solve a large number of carefully selected problems.

During the first half of the sophomore year algebra is again taken up. A thorough review of such portions of the elementary algebra as are deemed necessary by the instructor is followed by a course in advanced algebra. This course covers topics usually studied in the freshman year in the colleges and higher technical schools.

Five recitations a week during the second half of the sophomore year and the first half of the junior year are devoted to plane and solid geometry. The instruction aims primarily to use the subject as an instrument of education. Geometry contains a system of knowledge that is indispensable to success in many of the pursuits of life, but the presentation of this system of knowledge can never be other than a secondary object in a course of proper instruction in the subject. In reality the pupil ordinarily comes to the subject with many of its leading facts already in his possession. The real objects kept constantly in view in teaching the subjects are training in logical reasoning, an object of increased importance, as it is the only course in strict reasoning with which a large number of young people ever become closely acquainted; training in clear and accurate expression, an object not wisely neglected in any department of instruction; training in imagination and invention. To aid in these objects, extensive practice in original exercises is given, in which the pupil is required to devise his own proof, under the guidance and suggestions of the instructor.

The last half of the junior year is given to plane and analytical trigonometry. The textbook is supplemented by frequent familiar talks pointing out the best methods of procedure and illustrating the applications of the subject to surveying, navigation, etc. Special stress is placed upon the use of logarithms in computations and also upon analytical work to insure familiarity on the part of the pupil with the transformations and definitions necessary to success in future mathematical and engineering courses.

The schools considered are "secondary" schools, whatever their specific name, and consequently the aim of their instruction, except in the case of the private commercial schools and certain trades schools, is to a greater or less degree influenced by the requirements for admission to higher institutions. In fact, this element is an important consideration in all of the activities of these schools, though of course it is not necessarily predominant.

The mathematical curricula include arithmetic, commercial arithmetic, algebra, geometry, trigonometry, analytic geometry, the calculus, history of mathematics, and so-called "applied" or "shop" mathematics.

In schools where curricula are arranged with more or less reference to the entrance requirements of the colleges, algebra, geometry, and trigonometry receive about the same amount of time as in general secondary schools, viz, one year each of elementary algebra and plane geometry, and one-half year each of advanced algebra, solid geometry, and trigonometry. A year's work ordinarily represents five exercises per week for 33 to 40 weeks, i. e., a total of 165 to 200 exercises. The length of the exercises is from 40 to 50 minutes.

The data accessible render a precise statement of the division of time between the two algebraic subjects and between the two geometric subjects impracticable. The more advanced subjects are occasionally omitted, and in this case the tendency appears to be toward the retention of solid geometry and trigonometry rather than advanced algebra.

The schools to which the above statements apply are ordinarily, though not necessarily, of the type known as "manual-training" schools, as distinguished from the "technical," "industrial," or "trades" schools.

These latter schools, as well as some of the manual-training schools, prefer to offer a course in arithmetic of from 50 to 200 exercises.

A few schools present courses in so-called "shop mathematics," the nature of which appears from the following outline taken from the circular of a trades school:

COURSE I. ELEMENTARY.—SHOP ARITHMETIC.

This course comprises work with common and decimal fractions, measurements, percentage, ratio and proportion, square and cube root; applying these principles to such shop problems as gearing—simple and compound; how to select gears to cut screws and spirals; computations on the lever, including the lathe indicator, lever safety valve, the Prony brake; pulleys and hoists; simple, compound, and differential indexing with the milling machine; problems connected with the speed lathe and engine lathe; computing the horsepower of steam engines, electric dynamos, and motors.

COURSE II. ADVANCED.—ALGEBRA, GEOMETRY, AND TRIGONOMETRY, WITH APPLICATIONS TO SHOPWORK.

This course is open to those who have completed Course I or who have had a preparation equivalent to a good grammar-school education. It treats of the most important principles of *algebra*, especially of the equation as a means of solving problems and of the derivation and use of formulas. The practical side of *geometry* is next taken, emphasizing the methods of finding areas and volumes, weights of bars of various shapes and materials, heating surface of boilers, etc. The last half of this course is spent on *trigonometry*, including the use of logarithms and logarithmic tables and emphasizing the applications of trigonometry to the more advanced types of shop problems.

The private commercial schools give no mathematics but arithmetic and commercial arithmetic; the commercial departments of several secondary schools give commercial arithmetic and sometimes algebra and geometry; while the "high schools of commerce" always give commercial arithmetic and a course in algebra and offer geometry and trigonometry as elective subjects.

The agricultural schools generally give one-half to one year's work in arithmetic, usually with reference to the problems of farm life, e. g., farm accounts, mensuration, bookkeeping, etc. They ordinarily give the same amount of time to algebra and geometry as do other secondary schools. Advanced algebra is occasionally given, trigonometry in about 25 per cent of the schools, frequently with reference to its use in surveying.

Analytic geometry and the calculus are seldom given except in schools which properly belong in the province of Committee IX in that their work, while not leading to a degree, nevertheless covers the first two years of the work of the higher technical schools. Except for these schools, the subjects mentioned are offered only in courses preparatory to the colleges.

The history of mathematics is specifically mentioned by only one school (and that a school for girls) and is given in connection with the regular work in algebra and geometry.

As to the matter of correlation of the mathematical subjects among themselves or with other subjects, it would appear that it is necessary to distinguish between the actual state of affairs and the tendencies at work. Taking the schools as a whole, it may fairly be said that *systematic* correlation is not widespread. The principle of correlation, however, is generally regarded with favor, but as a rule it is not systematically applied, except where the relation of the subjects (e. g., commercial arithmetic and bookkeeping) is so obvious that the necessity is apparent. In so far as the term "correlation" indicates use of problems taken from the applied sciences or from daily life, there is fairly general application of the principle, but in the sense of systematic coadaptation of mathematical and other courses it is not generally applied.

The situation may be illustrated by the following quotations:

(1) On the technical side the pupil articulates the mathematics with the work of the drafting room, shop, domestic science, and domestic art. Teachers of technical subjects are in constant touch with the mathematics department, anticipating problems which will arise and reporting immediately to that department any weakness shown by a pupil in problem or principle.

(2) In this work a great deal of time will be spent in laboratory study, so that the pupil will obtain such a first-hand knowledge of the subject that he can afterwards readily and efficiently apply it in the shops and laboratories. No sharp distinction will be drawn between algebra, geometry, etc., but the different methods will be treated merely as various ways of getting at the same thing, of which one way may be the more useful in one case and another method that best adapted to deal with another situation. At all times the work in mathematics will be kept in close touch with the shopwork; the aim will be to so train the pupil that he can use his mathematics in the shop readily and efficiently.

These statements are made by schools which strongly emphasize the principle of correlation. Both are recently organized schools.

On the other hand, we have in answer to the question, "Are any systematic attempts made to correlate mathematics with other subjects?" the following:

(3) Some, but more later.

(4) In past, 25 cents on the dollar; will aim to do better in future.

In many cases the answer was a flat "No."

The chairman of the committee is of the opinion, based on internal contradictions in the evidence submitted by the schools, that no satisfactory conclusion as to the nature and results of the application of the principle of correlation can be obtained except on the basis of a study of a considerable number of schools, this study to be made by a single individual or a small committee and on the spot.

EXAMINATIONS.

There is no evidence to indicate that examinations are to any extent used as the sole means of determining the proficiency of the pupil. They are used as auxiliaries for that purpose, but the results are combined with those of daily work. They are ordinarily written, may be from 40 minutes to 4 hours in length, and occur from 30 to 2 times per year. The tendency is toward relatively frequent examinations, not exceeding 2 hours in length.

METHODS OF TEACHING.

The movement which has led to the establishment of the secondary technical schools finds its principal expression in the emphasis laid on the concrete. Consequently one should expect to find, and does find, that increasing attention is paid to the concrete element of the instruction, both in material and method. Nothing, however, could be further from the truth than an assertion that *as a class* those schools have developed and are using methods of instruction widely different from those of general secondary schools. Certain schools, it is true, have developed such methods; the majority have not.

The information derived from the questionnaires is not extensive or detailed enough to warrant detailed statements. The basis of the above assertions lies rather in the direct and indirect evidence contained in the catalogues of the schools.

As illustrations of such evidence the following statements are quoted:

(1) Pupils in mathematics are given acquaintance with the language of mathematical symbols, called formulæ, in which problems and laws involving weight, size, time, force, and the like are frequently stated. They are taught to understand these formulæ, to solve problems so stated, and to use the

mathematical symbols in the statement and solution of new problems. Pupils are taught also to state and solve problems by graphical methods, i. e., by scale drawing or by the *graph*, and immediately to solve the same by the algebraic methods of the equation or the proportion. The pupil becomes familiar with the standard geometrical forms, the laws of their structure, measurement, and relation to other forms, and acquires the power to state these laws algebraically, together with some ability to make a clear and logical proof of the truth of geometrical theorems. Geometry and algebra are carried along together for two years and a part of the third. In the first year the geometrical laws and concepts furnish much material for developing algebraic problems and processes. In the second year algebra is used to develop geometrical theorems, and to fix them in mind through use. The school offers a continuous four years' course of elementary and advanced algebra, plane and solid geometry, and plane trigonometry.

In the classroom a combination of laboratory, recitation, and examination methods is employed. The theory of a new subject, especially in the earlier years, is usually developed by the instructor; and home work is assigned to clarify and impress it, and to enlarge its application. The method of approach to new subject matter is, in general, that of induction, the particular leading to the general, the concrete to the abstract. Deductive work becomes more prominent in the late years.

(2) Throughout the entire course this study (mathematics) will be pursued as a means to quantitative determination in the workshop, laboratory, office, and countingroom. Much of the educational value lies in the grasp which is gives the students of quantitative relations.

Objective work will introduce new subjects, so that there may be a rational basis for intelligent use of symbols and a thorough conception of the power of the equation. Formulæ should be deduced from relations actually seen, so that the pupil may discriminate between the abstract formula and its concrete practical relations to real things.

The boys of our school will have several weeks of constructional geometry work at the beginning of their mechanical drawing. This helps to lay a good foundation for demonstrative geometry, as well as to be of great practical value in their future use of drawing.

Supplementary exercises are given to show some of the uses of algebra in the natural sciences. Correlation between algebra, geometry, and the sciences is shown wherever possible. The graph and some of its uses are taught in linear equations and in easy quadratic equations.

In geometry the pupil will to a considerable extent originate his demonstrations instead of simply *memorizing* those of the author. Model proofs will be given when necessary to teach good form and logical arguments, but as a rule such demonstrations will be given only when the pupils would otherwise be at a loss to know how to proceed.

When a class in trigonometry has developed the working formulas it does most of its problem work in the field with the transit, leveling rod, and tape-line. Much of the work is plotted to scale. This work is found to be interesting and practical.

FIRST YEAR.

First term. Algebra to simple equations, including the application of factoring in simplifying fractions, and in solving easy quadratic equations of one unknown quantity.

Second term. Algebra to ratio, including easy exercises taken from the physical and chemical laboratories. The simpler uses of the graph will be taught during this term.

SECOND YEAR.

First term. The first two books of plane geometry. Special attention will be given as to what constitutes a rigid proof. Suggestions are given on methods of attacking propositions and problems. Neat, accurate form work will receive special attention, and pupils will be required to bisect lines, angles, erect perpendiculars, and draw parallel lines by *actually using compasses and ruler.*

Second term. Plane geometry completed. We expect most pupils by this time to be able to do considerable work on their own initiative; to be able to have some determination to master a proposition set before them.

The practical applications of the subject are shown whenever possible to do so.

THIRD YEAR.

First term. Solid geometry.

Second term. Elementary algebra completed. This includes ratio, proportion, variation, imaginaries, series, partial treatment of binomial theorem, logarithms, review.

FOURTH YEAR.

First term. (1) Plane trigonometry. Development of formulas. Fieldwork with transit, leveling rod, and tapeline.

(2) Higher algebra.

Second term. (1) Descriptive astronomy. A brief, simple, and accurate account of the heavens as they are known to-day. It is intended only for high-school pupils, and to give some information on the subject that is needed for the person of ordinary culture. Interesting facts will be studied, but no attempt can be made to gain any clear conception of the processes by which the fundamental truths of astronomy have been established. The methods of discovering the wonderful truths of this subject belong to the advanced student.

(2) Higher arithmetic for those who expect to become teachers.

NOTE.—The formation of these classes depends on the number of pupils who can arrange their programs so that it would justify the taking of a teacher's time from the crowded classes in first, second, and third years.

These quotations indicate what is done in certain schools which have paid much attention to the development of methods.

On the other hand, there are two sources of evidence for the statement that the schools *as a class* have not developed special methods of instruction. The one is the negative answer in the answers to the questionnaire; the other is in the lists of textbooks. The use of texts dominates the instruction in the majority of American schools, and hence the nature of the text is to some extent an index of the character of the teaching. When one finds, as is actually the case, a widespread use of a few texts in which the treatment is essentially abstract and the problems constructed with small reference to other than formal requirements, it is a justifiable inference in the absence of evidence to the contrary that the instruction in the schools using these texts is not markedly different from that in general secondary schools using the same texts, and this inference is strengthened by detailed statements of matter covered, which not infrequently are extracts from the tables of contents of these texts.

While the use of texts, and, indeed, of traditional texts, is general, the newer schools, especially those in which the industrial idea is prominent, are to some extent breaking away from these texts and the corresponding instructional methods. A number of schools have prepared collections of problems taken from shop practice or from matter contained in technical periodical literature. Unfortunately but few of these are accessible in published form.

As to the process of instruction, it would appear that the method of holding recitations upon assigned textbook matter is not extinct. This method, which renders the recitation merely an oral examination, is, however, giving place to the method of development of new matter by questioning on the basis of the pupil's fund of knowledge. The use of problems as an instruction to new matters of theory is apparently more common than the use of problems solely as applications of a didactically presented theory.

It must be said, however, that the evidence on this point is meager and conflicting.

It must be borne in mind, moreover, that even approximate uniformity of method does not exist. The nature of the school, the necessity of preparing students for external examinations, the preparation and personality of the teacher—all have their influence on the method of instruction.

The laboratory method of instruction can not be said to be widely used. The term may be taken as denoting the use of experimental processes devised for the purpose of discovery or emphasis of mathematical truths. The mathematics is the final goal, the physical process the means. The method may be illustrated by the following quotation:

> Algebraic problems are developed from the laws of percentage from the sides, angles, and areas of polygons. The laws of the lever and of beams are established by experiments in the classroom and are made the basis for the development of the fundamental processes and the laws of sight. Drawing to scale gives many problems in similarity of triangles and in ratio and proportion.

Allied to the laboratory method, but distinct from it, is that which may be termed the "shop method." Here as in the former physical processes are used, but the physical result is the final goal, the mathematical truth, a thing which is introduced and developed because it becomes necessary to the accomplishment of that end. The method may be illustrated by the following quotation:

> WORKSHOP MATHEMATICS.
>
> After a thorough review, which demonstrates to the pupil and the instructor the ability of the former for this important branch of his trade, the apprentice is led, by the solution of practical problems, through the necessary portions of arithmetic, algebra, geometry, and trigonometry. These subjects, when presented to pupils in the abstract, are frequently beyond their mental grasp, but when connected with their trade practice the absolute necessity of this knowledge becomes plain; the student then attacks the problem from a new standpoint and with renewed vigor, and succeeds in mastering the difficulties.

All the problems in this branch of apprenticeship, also, are specially prepared by the instructors and printed by neostyle. Much of this work is required to be done by the students as home study. Lectures and shop talks supplement the workshop mathematics.

It is, of course, scarcely necessary to add that the distinction between the two methods, as they are actually used, is not absolute. It is a difference in the position of the center of gravity of the instruction, which, if great, may amount to a qualitative distinction.

Some of the more distinctively vocational schools and all the commercial schools emphasize to a greater extent than the others the matter of computation, for the evident reason that with them the numerical result is a matter of technical importance.

In the commercial school particularly unremitting drill on the elementary processes of arithmetic is an essential feature of the instruction. Naturally the proportion of time expended on such drill depends on the breadth of the curriculum, and is greatest in the private commercial schools, where the only mathematical subject, commercial arithmetic, is essentially a technical subject. In the "high schools of commerce" the drill is important, but the broader curriculum permits emphasis on the theoretical side of the subject. Naturally, in all the commercial schools much attention is paid to the use of material drawn from commercial practice.

PREPARATION OF CANDIDATES FOR TEACHING.

In the greater number of schools a considerable proportion of the teachers are graduates of "normal schools," or schools of college grade. In many instances they are graduates of engineering schools, and, in a few cases, they possess the doctor's degree (obtained in course). In the trades and industrial schools some of the teachers have had experience in a trade or in one of the engineering professions; in the commercial schools many of the teachers have had experience in business houses.

The question of the preparation of teachers is the gravest which these schools have to face, particularly those in which the trades or industrial element is predominant. "Normal" courses for teachers of "manual training" exist, but there appears to be as yet little provision for the training of men who, with an adequate knowledge of the technique and problems of a trade, also possess a thorough knowledge of the science of mathematics and of the theory and practice of education. Such men exist, but they are few in number[1] and are the result of accidental circumstances, not of organized instruction. A few institutions, notably Teachers College, Columbia University, are now offering courses designed to meet the needs of persons preparing to teach mathematics in the secondary technical schools.

PART II.

MODERN IDEAS CONCERNING SCHOOL ORGANIZATION.

The manual training, the industrial, the trades, the agricultural schools, and the high schools of commerce are, in their present form, essentially new types of school which have been developed as the result of the movement to render instruction more concrete and immediately available.

COEDUCATION.

The schools considered by this committee are fortunate in that the vexed question of coeducation presents itself in so objective a manner as to permit sane discussion, in some respects at least. The question of the simultaneous attendance of the two sexes at the same institution is one which may be regarded as settled in America by custom. This question need not concern us here.

On the other hand, in proportion as the schools are distinctively vocational, the question of segregation rather than coeducation becomes important. Segregation becomes imperative when the vocational element is predominant and differs for the two sexes. Whenever in the schools considered, correlation of the mathematical instruction with that in the technical subjects is regarded as essential, the difference between the technical interests of the two sexes is found to be so great that the successful application of the principle of correlation renders segregation necessary.

On the other hand, in the commercial and agricultural schools the interests of the two sexes are so nearly identical that segregation in the classes in mathematics is neither imperative nor usual. The same considerations hold with reference to the sex of the teacher.

MODERN TENDENCIES CONCERNING THE AIM OF INSTRUCTION AND OF THE BRANCHES OF STUDY.

There is a tendency to omit so-called "useless subjects," but the criteria are variable and often contradictory. The tendency, however, is to omit subjects regarded as involving complex manipulation or difficult theory unless they are of essential and immediate vocational importance. For example, the extraction of numerical cube root, partial payments, etc., are omitted from the courses in arithmetic given in the agricultural schools. On the other hand, the commercial department of a high school omits as useless the subject of graphs from its course in algebra. In general, however, the tendency is to retain the traditional content of the courses of the general secondary school, except in those schools whose courses are arranged with reference to immediate technical availability.

The general tendency is, moreover, not to increase the content of the courses or to replace old subjects by new except in so far as the so-called "workshop mathematics" may be regarded as a new subject. Such courses are new in their point of view and their concrete material, but not in their mathematical elements.

In this connection may be mentioned, however, the general tendency to introduce elementary trigonometry into the curriculum because of its numerous applications to shop problems and to surveying.

EXAMINATIONS.

There appears to be no noticeable tendency to abolish examinations but rather to subordinate them to the regular work.

METHODS OF TEACHING.

With the increase in the size of the cities and the centralization of industries has come a decrease in the fund of general knowledge which is available in the child as a basis for mathematical instruction. In the days of the small shop in the small town the artisan's boy frequented the shop and used his father's tools. He learned in a desultory and accidental way, perhaps, but nevertheless he learned to plan, measure, and build, and became acquainted with the materials and methods of the various forms of industrial activity. From all this the city boy of to-day is excluded.

In consequence of this the concrete basis which the boy formerly obtained for himself, and unconsciously, must now be systematically provided by the school. The mathematically ill-equipped teacher of an earlier period had no suitable mathematical superstructure to erect on the excellent foundation provided by the boy; the better-equipped teacher of to-day finds the foundation inadequate to support the structure he is prepared to erect. The consequence of this essentially economic change is that systematic intuitive instruction in the school is becoming increasingly necessary and is supplied with increasing frequency, e. g., by the use of "laboratory" and "shop methods" and by the use of concrete problems.

The technical secondary schools from their very nature tend to adapt their instruction to this need more readily than do the general secondary schools, and with apparent success. One school reports:

> The boys take hold of mathematics as if they "needed it in their business." Our boys are not exceptionally bright, and yet the best section of girls in a "regular" high school would not be able to keep pace with them.

Along with this tendency to emphasize the intuitive element in instruction, there is the conservative tendency, partly due to inertia and partly to external academic requirements, to emphasize the abstract element.

In view of the lack of central control of American schools, the existence of these two conflicting tendencies, especially the latter, can not be regarded as otherwise than fortunate for the future of mathematical study and research in this country.

The general secondary schools already feel the competition of the technical secondary schools. The danger lies in the possibility that the attractiveness of the intuitional and immediately available element of the instruction in these technical schools, whose ideal approximates to that of the trades school, may so diminish the abstract and logical element in the mathematical curricula of all the secondary schools, general as well as technical, as to hinder the progress of mathematics as a science in this country.

The lack of concrete basis for mathematical instruction is the cause of a tendency, expressed in the curricula of certain schools, to precede the demonstrative work in geometry by work in "constructive" or "inventional" geometry. The circular of one school says:

> This study (geometry) is, first of all, inventional. With ruler, dividers, compass, and protractor, the pupil is taught to draw geometrical figures and then to study and understand them. After a term's work in industrial geometry, the pupil studies plane and then solid geometry with an interest which would not be possible otherwise.

The age of the pupils in the schools considered is such that, once they have become familiar with the elementary geometric concepts through their work in constructive geometry, the logical element of the subject can be made predominant.

Material aids to mathematical instruction are of course much in evidence in schools which make use of the "laboratory" or the "shop method" of instruction. In geometry and trigonometry, when climatic conditions are favorable, outdoor work is occasionally used. In solid geometry the use of models made by the pupils is not infrequent.

Excepting in the commercial schools, where the matter is one of vital importance, the matter of computation does not receive the *general* attention one would expect in schools of essentially vocational purpose. A few schools, however, lay much stress on the matter, and one, at least, offers a course on the subject, the outline of which may be quoted:

> COMPUTATION. B class. (Applied Mathematics.)
>
> A course in the interpretation and application of standard engineering formulæ, abbreviated methods of calculation, the use of mathematical tables, approximation by graphical methods, and the use of computing devices. The solution of practical problems.

The aims of the course are:

1. To give the student some adequate acquaintance with computing methods.
2. To develop in him at the same time accuracy and speed.
3. To cultivate the ability to estimate results with a reasonably close degree of approximation.
4. To minimize labor in his calculations.

The experience of the vocational schools indicates that the solution of the problem of giving mathematics a better place in popular instruction and of reacting against the prejudices against the science lies in bringing the subject into such close relation with the activities of daily life, especially those of an industrial nature, that the necessity of a knowledge of the subject is felt. From the standpoint of the progress of the science this is the valuable element in "workshop mathematics."

The dangerous element in "workshop mathematics" is not fundamentally distinct from that which is so often the bane of abstract instruction and against which the representatives of the secondary technical schools so emphatically protest, namely, excessive formalism. By formalism is here meant the unthinking and mechanical execution of mathematical processes without regard to the significance of the data, the operations, or the results.

This formalism may be illustrated by the following problem taken from a published collection that is somewhat widely used: The dimensions of the parts of a rather complex combination of crank, screw, and gears are given, and it is required to find the weight which can be raised by a force of 60 pounds applied to the crank. The mathematical work involved is merely the numerical evaluation of a rational fraction the factors of whose terms are given. The published answer is 203,575.68 pounds; that is, a weight of a hundred tons is given to the sixth part of an *ounce*. It is evident that such a result can be obtained only by a mechanical and unthinking use of the mathematical processes involved, and without the slightest consideration of the significance of the concrete elements of the problem. This is formalism pure and simple, and it is the more pernicious in that it masquerades under the guise of "shop mathematics" and claims to be an example of "how to apply mathematical principles, rules, and formulas to the solution of such (i. e., shop) problems."

There is a desire among some of the teachers of mathematics in the secondary technical schools to break down the conventional barriers between the several branches of mathematics, and in a few cases this desire has been realized. The principal difficulty mentioned concerning this movement appears to be the lack of textbooks designed for such combined courses, a difficulty which is not inconsiderable by reason of the dominant position of the textbook in American mathematical instruction. A few such texts, however, have been published.

There are administrative difficulties, particularly in the case of schools which articulate closely with the grades on the one hand and the colleges on the other, and these difficulties, while not mentioned by the schools, have proved a serious barrier to the movement in the colleges. The conservatism of teachers also retards the establishment of these combined courses.

It would appear that the technical and industrial schools offer unusual advantages for the development of combined courses, because of the fact that, in the newer ones at least, the articulation with established schools is less close and the force of tradition less strong than in the general secondary schools. It must be remembered, however, that these combined courses are of comparatively recent origin, while the traditional courses are the result of a long period of evolution. The combined course is in the experimental stage, and for this stage of its evolution the secondary technical schools offer, for the reasons just mentioned, a favorable culture medium.

Wherever the courses are separated it appears that the study of the elements of algebra precedes that of plane geometry. The relative position of the second course in algebra and that in solid geometry is variable.

RELATION BETWEEN MATHEMATICS AND OTHER BRANCHES.

While *systematic* coadaptation of the courses in mathematics and those in other subjects is not general, there is a strong tendency to make such adaptation to a greater or less degree. The tendency may, and sometimes does, take the form of emphasis on the application of mathematical results at the expense of the logical and demonstrative element of mathematics and of its dignity as an independent science. The "pocketbook engineer" has his counterpart in the secondary schools. More frequently, however, the tendency finds a more rational expression in the form of emphasis, in the mathematical courses, upon problems derived from other branches. These problems may serve as an introduction to the demonstrative work or as an application of its results.

The difficulties and dangers in the working out of this tendency are precisely the same as those which arise in a similar situation in the higher schools. Unfortunately, neither the teacher of mathematics nor the teacher of the technical subjects is omniscient. The one lacks technical training, the other a thoroughly grounded knowledge of the science of mathematics.

A study of published collections of problems used by some of the schools indicates that the mathematical principles involved in the technical problems considered in their courses are, for the most part, of a very elementary character. In geometry, the propositions of congruence and similarity, the theorem of Pythagoras, and the mensurational theorems; in trigonometry, the definitions and elementary properties of the functions with their use in the composition and resolution of vectors; and in algebra the fundamental operations and the solution of linear equations and binominal equations of lower degree form the theoretical basis for the greater part of the problems in question. The problems arising in surveying, of course, require more extended knowledge of trigonometry, and the varied problems of the machine shop involve algebraic principles of a more advanced character.

For example, problems on the efficiency of hoisting devices (friction considered) and in the design of cone pulleys involve geometric progressions; problems of gearing and screw cutting involve indeterminate equations, Euclid's algorithm of the greatest common divisor and continued fractions.

REPORT OF SUBCOMMITTEE ON SECONDARY COMMERCIAL SCHOOLS.

Sources of information.—This report is based on statistical information obtained by means of questionnaires and on other data available to the members of the committee, especially the chairman, as members of the instructing staffs of commercial schools.

The report was prepared by the chairman of the subcommittee in consultation with the other members and with the chairman of the committee on secondary technical schools.

Aim of the report.—As some of the work of the schools considered does not greatly differ from that of general secondary schools the subcommittee has confined itself largely in this report to the consideration of the points of difference between the work of the commercial and of the general secondary schools—the nature, cause, and results of these differences.

Classification of schools.—The schools considered by the subcommittee fall into three classes, viz. high schools of commerce, commercial departments of general secondary schools, and private commercial schools (the so-called business colleges).

The private schools were first in the field; their primary aim was and is preparation for immediate vocational activity. Though they are, therefore, essentially of the same nature as "trades schools" and are largely conducted for profit, the committee is impressed with their educational value and the professional spirit of their instructing staffs.

There are many of these schools throughout the country. The fact of their existence is proof of the demand for the kind of education they offer. It is a further testimony of the work done by these schools when it is cited that some cities, as Berkeley, Cal., already offer a two-years' course in commercial subjects. Boston, Mass., has just voted to establish a central clerical high school, to be in session from 9 to 5; its scope of work will be that usually offered by the best business colleges.

The commercial department in the general high school is the natural outgrowth of the success of the private commercial school, just as earlier in the history of our country the general high school was the outcome of the success of the academy. Moreover, just as the private academies began to go out of existence with the success of the general high school, so that now but a comparatively few of those in existence from 1850 to 1860 still remain, in the same way it is probable that the public schools will do more and more of the work now being done by the private commercial schools. And just as there are now some academies still in existence, and in a most healthy condition, so is it likely that we shall always have our private commercial school; but it is improbable that we shall have all that we have at present.

The same observation which is made here with regard to the pioneer work of the "commercial college" is manifest in the history of many features of our present educational system; the need is first shown by experiment carried by private enterprise, either philanthropic or commercial; then the public-school department, hitherto passive, becomes eager to incorporate the private success into its own field of activities.

Since the commercial department of the general high school aims to do the work of the private commercial school, there will be but little to report in regard to them in addition to what will be reported for the private school beyond the fundamental differences between the two kinds of schools in all respects.

The commercial high school is but one step beyond the commercial department and must be from the nature of affairs restricted to the larger cities. However, this step to the separate high school is a long one. The "high school of commerce" expects to give a better fitting for business life than either the private commercial school or the commercial department can: better than the one because the course is longer and its scope is broader: better than the other because the work of the four years in the high school is much more specialized.

COURSES OF THE SEVERAL TYPES OF SCHOOLS.

In the private commercial school the length of course varies from three months to two years, depending on the preparation and wish of the pupils. The usual course, however, is for one year. Certificates or diplomas are given for work covered. The same work is given for both boys and girls, as would be expected since they are found in the same class. The entrance requirements are not as clearly defined as for the public secondary school; the scheme is rather to put the pupils into those classes where they can take up the work to best advantage. The age of the pupils varies from 14 to 20, as a general rule; both younger and older pupils, however, will be found in attendance.

In the commercial department of the high school, the course is of four years' length; the studies in the commercial department are but a part of the student's studies; the others are taken from the general course of the school. The diploma given at the end of the course is the general school diploma and does not usually specify that the pupil is a graduate of the commercial department; it states rather that he is a graduate of the school; the diploma is the same as that given to the graduates of all the other departments of the school—this statement holds true so far as facts have come to the observation of the members of this committee. The work of the first year for pupils in this course does not vary materially from that of the other pupils of the school; accordingly, the same entrance requirement holds for all—that they satisfactorily complete the grammar-school course. The average age is from 14 to 20 years from entrance to graduation.

In the commercial high school, usually called "high school of commerce," the same general consideration for age and entrance holds true as in the case of the commercial department. The curriculum, however, is more specialized. The aim of the school is to prepare for a commercial life in a broad sense. The commercial departments prepare more for secretarial and clerkship positions and make bookkeeping, stenography, and typewriting the courses around which the work of the school centers. The schools of commerce make the economic sciences and courses the subjects around which the work of the school centers. The other subjects are studied, but are given but comparatively small emphasis in working out the aims of the school. Business men's organizations connected with such schools are of great value to them.

THE AIM AND SCOPE OF THE MATHEMATICAL INSTRUCTION.

The only branch of mathematics that is taught in the private commercial school is commercial arithmetic—sometimes called business arithmetic. The placing of commercial arithmetic as one of the branches of mathematics is one of the points of difference between the American and the German practice, for in Germany it is considered as one of the branches of the commercial studies. The aim of the work is to give drill, constant drill, in the ordinary operations of business, so as to secure habits of accuracy, speed of computation, and skill in mental operations.

There are usually five recitations per week, and the usual length of the recitation is 45 minutes. Some schools have recitations, however, that are one hour in length. The work in commercial arithmetic in these schools is made to correlate more with the work in bookkeeping than with any other subject. The teachers in these schools frequently make the complaint about the previous preparation of their pupils in arithmetic, that they lack in accuracy, speed, and knowledge of practical problems. Some schools report, however, that the foundation which they have in mathematical training is excellent for the drill that the business college wishes to give them. The comment made by the teachers in these schools is also made by many of the teachers in the other types of schools.

In the commercial departments the same subject is offered as is offered in the private commercial school—commercial arithmetic. The subject is nearly always elective in the school, but required for those who wish to take the commercial course. The length of recitation period varies from 40 to 45 minutes usually, and the number of recitation periods varies from two to five per week throughout the year, or the equivalent of it. The work is most often taken in the second of the four years of the pupil's course in the high school. It is sometimes, however, taken up in the first year of the course. In schools where the pupils also take algebra either the arithmetic is carried on at the same time that the algebra is or it is taken up after the pupils have had one year of algebra. The latter is more often the case. Accordingly, the usual age of pupils taking this subject is 14 to 16 years. In the New York High School of Commerce the subject is given by the commercial department; in Boston by the mathematical department.

In such schools as these the commercial arithmetic is usually correlated with the work in bookkeeping; less often it is correlated with work in physics. One school reports that it is made to correlate with the study of commercial law. It is very evident where this can be done, since the discussion of many problems arising in commercial arithmetic involve questions of both custom and law.

Where algebra is offered in these schools the pupils of the commercial departments are found in the same classes that the pupils of other departments are found in. Hence the report given by the committee on the general high school as regards algebra will be applicable to the algebra of the commercial departments. This same observation holds true for geometry and trigonometry.

In the commercial high school, however, mathematics is given more time. In the first year algebra is taken up for either four or five periods per week, with recitation periods of from 40 to 45 minutes in length. The algebra is offered largely as a vocational subject, and the problem is made the central thought in the development of the subject; the equation is studied in so far as it will help in solving the problem, and the different processes are taught only in so far as they help in solving the equation. The general aim is to study the applications, and only such principles of the science as help one to understand the applications.

The work in this subject is correlated with the other work of the school. It is evident where it is correlated with arithmetic, from the close relation existing in the nature of the subject, it is correlated with the work in science, in drill work, in the formulas of physics, and in problems taken from the field of natural science. In the use of squared paper and in the graphical representation of simple statistics, it is correlated well with the work in commercial geography and economics; in the matter of requiring the same standards of good penmanship that are exacted in the penmanship classes and in the matter of rapid calculations, invoicing, and the various applications of percentage, it is correlated with the business technique classes; in requiring explanations, oral and written, expressed in good English, it is correlated with the work of the English department.

The description here given for the method of teaching algebra is taken from the usage of the Boston High School of Commerce; it applies with equal force to a great many general high schools of t country. It would be very hard to present a statement about th teaching of algebra that would be above censure; some criticism would be sure to come from some part of the country. There is a feeling of unrest throughout the country as regards the teaching of algebra, and eagerness to make the subject more significant than it has yet been in secondary instruction.

The work in commercial arithmetic has all the features of the same work done in the private commercial schools and in the commercial departments. The time allowance is the same as for algebra. Since the merchants are much interested in these schools, practical material is available from them which would be otherwise impossible to obtain. The stores of the city are the laboratories of the students of the commercial high school. The books and accounts of such stores have much material that is of the utmost value to the school, and that material is being sought out by the teachers of the commercial high school, and already quite a little has been brought into use in the classroom.

Besides the drill in rapid and accurate performance of arithmetical operations, this work aims to give drill in the real significance of numbers, drill in habits of sustained attention and concentration of effort, and drill in specific problems brought in from the business houses. The general aim is to give drill rather than to teach the pupils anything new. The subjects emphasized are the fundamental operations applied to integers, common fractions and decimal fractions, percentage, interest, stocks, bonds, sinking funds, insurance and taxes, and equation of accounts. Arithmetical operations which are seldom found in business are omitted.

This work is correlated with other departments of the school in many ways. The extensive study of significant statistics having to do with the commercial life of the city, State, or Nation, and the graphical representation of the same in all the more common forms of graphics are of great value to the pupils in their economic studies later on, as is likewise the study of actual problems in stocks, bonds, and sinking funds. The teachers of bookkeeping and commercial arithmetic are constantly in communication, so that their work may be of much assistance to each other. The work of the English department is emphasized as in the case of algebra. The work is correlated with the science department in the manipulation of some of the practical problems encountered in the science work.

Geometry and trigonometry are sometimes studied, but they are studied for their cultural rather than for their vocational values; accordingly there is a freedom of method and choice of theorems which would otherwise be impossible.

It is pointed out that a school which emphasizes the practical side of algebra should also emphasize the practical side of geometry and trigonometry. This statement is true, and it is quite probable that some time in the future we shall have a course in these studies more applicable to the commercial high school. This, however, is a problem which has not yet been worked out in connection with so comparatively recent a departure in education as the commercial high school.

The preparation in mathematics of pupils entering the commercial high school is the same as that of those who enter the classical high school or the general high school. Since these features of the American educational system should be more properly treated in the reports concerning those schools, they will be omitted here.

In both the private commercial school and the commercial department, special desks are provided for the students; these desks are made to conform as much as possible to commercial usage. In the commercial high school these desks are found in the department of business technique.

In some schools which have no commercial department a half year of advanced arithmetic or commercial arithmetic is offered during the senior year, less often during the junior year; this course covers such essentials of everyday arithmetic as every high-school graduate is supposed to be prepared with.

THE EXAMINATIONS.

In most States diplomas from the school are not dependent on a final examination of the work covered during the course, nor are the examinations at all of that nature. They are more commonly given every week, two weeks, or month on the work covered during the time since the last examination. The examinations are only incidents of more or less importance in the progress of the work.

The regular daily work is considered of much more importance in estimating the pupils' progress and educational worth than the examinations are. This principle holds true of all three kinds of schools.

THE METHODS OF TEACHING.

Textbooks are very freely used both for class work and for home work: and many excellent books on commercial arithmetic have been written by the teachers in these schools, particularly by the teachers of the private commercial schools. Since the general aim of the work is to give accuracy and speed, much oral work is given for mental drill. The textbooks are made up of the kind of problems that are met with in the business activity of the country as far as the author has discovered usage and custom of business houses.

Interest tables, simple and compound, are the only kind of tables that find much use in the arithmetic work; a few schools report the use of logarithmic tables. The Boston High School of Commerce advocates their use in the commercial arithmetic work, by the observation of their use in statistical offices by people who have never studied any mathematics beyond the arithmetic.

Many of the private schools, fewer of the public schools, report the possession of adding machines. This is not surprising when we consider how recently such machines have come into general commercial use. It is significant, however, that our schools are beginning to recognize that the adding machine is as necessary to modern business as is the typewriter. Instruction on the latter forms a recognized part of every commercial course, and within a few years the skillful operation of the adding machine will doubtless be taught in all the well-equipped commercial schools.

One school reports the possession of a cash register and loose-leaf ledger. Other kinds of calculating machines are seldom, if ever, found in the schools.

Some teachers in the commercial high school report the use of the laboratory method to good advantage. By this method a set of exercises is chosen, in some such subject as common fractions, from the commercial life of the city, State, or Nation. Each pupil is given a separate problem—an experiment—on which to work. Inasmuch as all problems are different, the pupil is thrown on his own resources; since results are proportionate to efforts expended, such work offers an incentive to ambitious pupils for extra work.

Sets of exercises have been compiled for such subjects as addition and subtraction, multiplication and division, common fractions, decimal fractions and percentage, exchange work, stock invoicing, marking goods, and sinking funds.

The methods in algebra vary from the usual course in that subject as much as is consistent with the different aim of the subject; the mental exercises are more particularly drilled in the commercial high school than in the general high school. Ratio and proportion are taken up early in the course, since the work is so closely related to the work in equations.

Geometry is taught quite irrespective of college entrance requirements.

PREPARATION OF CANDIDATES FOR TEACHING.

But a few of the teachers in private commercial schools are college men. The preparation of the teachers for the work of instruction has been by attending the private commercial school, or by having attended a commercial department in a general high school, or by having had actual business experience. Many of the teachers in these schools have had this business training, and, of course, this fact gives them one excellent qualification for this particular kind of instruction. Teachers are quite often found in these schools who have had practical training in higher accounting or have pursued courses in the subject.

The preparation of teachers for the commercial high school is of higher grade, from an academic point of view, than that of teachers of the other two departments. The best teachers for this particular kind of a school are yet to come. The school is a comparatively recent innovation; accordingly, the demand for teachers for this special kind of school is comparatively a new one. They nearly all now have good academic training; what they will need in the future is more business training. The teachers are taking the problem seriously, too, and are making decided and effective steps to supply their deficiency in these regards.

The preparation of the teachers in the commercial department of the high schools, of necessity from the nature of affairs, has been much the same as that for private commercial schools. Preparation has been mainly in the private commercial schools and from the commercial departments of the high schools. A smaller number by far have obtained business experience; this is probably due to the lack of salary inducement more than to the nature of the problem.

In the past, few college men, or women, have gone into this line of professional work; but with the present tendency toward vocational education, so manifest in the country, more college men are undertaking it.

A marked advance was made two years ago when the Salem (Mass.) State Normal School offered both pedagogical and vocational courses in the commercial subjects. This course has met with such success since its institution that the course has been lengthened to three years, with a corresponding addition of opportunities offered to the students. A few other State normal schools in the country had previously offered the vocational courses. The Albany (N. Y.) State

Normal College has since then offered the same kind of courses that the Salem school offers. The private normal school at Valparaiso, Ind., has long conducted training courses for commercial teachers, as have also several other private normal schools of the Middle West. It is further interesting to note that the Teachers College of New York City is preparing to inaugurate (1910–11) a course in commercial arithmetic, and later one in stenography and typewriting. Simmons College for Girls, of Boston, for the past few years has also been meeting the demand for instruction suitable for preparation for teachers of commercial departments of high schools.

It can readily be seen that a great many teachers of commercial subjects in the future will get their preparation from these normal schools. It is well that these schools offer this opportunity, for the demand for commercial education becomes more pronounced year by year, and there is constant demand for more and better trained teachers.

REPORT OF THE SUBCOMMITTEE ON SECONDARY AGRICULTURAL SCHOOLS.

The agricultural high school is of recent origin in the United States and owes its existence to several causes. Probably those most potent are the growing conviction in the mind of the rural community that it must furnish some form of education beyond that given in the district school of grammar grade, the recent awakening to the necessity of industrial education of some form for every community, and the attempt to induce the General Government to appropriate funds toward the support of agricultural high schools in each congressional district.

That the concentration of school districts in the country, with provision for the transportation of pupils from a sufficiently large territory to form a graded school, has materially increased the efficiency of the rural schools is undisputed. That the same principle extended to the secondary school will be successful is expected by the rural communities which have undertaken the task of secondary education.

These secondary schools, started in agricultural districts and maintained in practically every instance by public funds, in many cases by the State itself, have attempted with more or less success to adapt the work of the school to the principal industry of the section—that of agriculture, and accordingly they have formed their curriculum to give prominence to the sciences underlying that industry and have curtailed the instruction in other lines, most noticeably perhaps that of foreign language.

This brief statement is made by the subcommittee in order that the result of the questionnaire sent out may be better understood.

Your subcommittee sent to the department of public instruction of nearly every State a request for the list of secondary agricultural schools within the State, and in practically every instance a reply was received giving the information asked for. In many instances schools not of secondary grade were mentioned, as well as schools in which "some" agriculture was taught, but which were not styled agricultural schools. The subcommittee, believing that an inquiry into the teaching of such schools was not within their province, rejected all schools mentioned which did not seem to come under the head of secondary agricultural schools. As a result of the inquiry, a list of 65 schools was obtained which seemed to embrace all that properly came under the class delegated to this subcommittee.

The distribution of these schools was of interest to the subcommittee. About half the number were in the Southern States, and nearly as many in the middle Western States, while the East and Far West were represented by an occasional school. Some of the leading agricultural States seemed to have no schools of this kind, notably Iowa, Illinois, Missouri, and Kansas. It was found that schools of this class existed in only about one-fourth of the States, and that in but 8 States, of which 5 were Southern States, did they seem to be established by districts.

A questionnaire of limited scope and admitting of easy reply, covering the source of students, method of support, kind of control, special aim of school, general and mathematical curriculum, methods of admission, and special questions on the instruction in mathematics, was framed and sent to the schools listed, and replies were received from about 40 per cent of the number, and judging from the replies, in most instances from typical schools of the different sections. Based on these replies, the following deductions and comments are given:

The schools are generally located in the country, or in small rural towns, and draw their students largely from the homes of the farmers, although a number report that the students are recruited from all classes. They are all supported by public funds. Some have State support and others are maintained by local taxation. In most instances both State and local funds are used. They are practically all coeducational.

The predominant aim seems to be to fit the pupil for farm life, and to emphasize those subjects bearing on agriculture and domestic science. In many instances the avowed purposes of the school are to create a deeper interest in things agricultural, to direct the interest of the pupil to the problems of the farm, and to hold the attention to the advantages of rural life as compared with those of the city.

To attain these ends the curriculum is more or less affected. Special stress is laid upon the sciences, and departments bearing directly on agriculture and domestic science are introduced. A number report that the curriculum is not materially affected, some that it is more practical or more scientific, others that it is industrial, and one that all subjects are taught from the standpoint of the farmer. To provide for the vocational subjects in many instances the amount of foreign language taught has been curtailed.

The mathematical curriculum is reported by many schools as not at all affected by the special object of the school. A few report that the amount of mathematics is reduced. One reports more geometry and surveying than would otherwise be given. Quite a number report that the work is more practical and that advanced arithmetic and farm accounts are included.

The replies to the question asking how the specific aim and object of the school affected the method of teaching mathematics were so diversified that no general conclusion can be given. About a third of the replies state that the methods are more practical, but only a few state in what way—that is, whether in subject matter and character of problems or in methods of teaching. About a fourth of the replies state that the method of teaching the subjects is not affected. Several make no reply, probably because the methods are unaffected. One states that the laboratory method is used and one that the work is mostly individual on account of the uneven preparation.

The subject of the correlation of the various mathematical subjects seems to have received but little attention, and but few report any effort in that direction. No attempt seems to be made to correlate mathematics with agriculture, except in the case of arithmetic. A few report that an attempt is made to correlate with physics and with science. In general a favorable opinion of correlation is expressed, although it appears that very little attention is given to the matter.

The use of the laboratory method with mathematical subjects seems in general to be very limited. Where used the reports state that the results are good.

In general the requirements for entrance are the completion of the work of the common school, which varies somewhat in the different localities. Eighth grade arithmetic seems to be about the maximum requirement in mathematics. One reports no entrance requirement excepting a minimum age limit of 14 years for boys and 13 years for girls.

Entrance is generally secured by either examination or certificate, although a few admit by examination only. The work in mathematics in the different schools, while showing a considerable variation, differs more in degree than in subject matter. A majority of the schools present arithmetic of some sort, under the name of practical arithmetic, farm accounts, bookkeeping, mensuration, etc. Practically all include a reasonably good course in algebra. Well-known texts are used and a fair amount of time allotted to the subject. The same may be said of the course in plane geometry, while a number pay attention to the applications as well. Solid geometry is given in quite a large number of the schools, and the time allotted to the subject seems sufficient to do the work satisfactorily.

In about 25 per cent of the schools replying trigonometry is given, and in a somewhat smaller number, plane surveying.

The relative stress placed upon mathematical dexterity, analytical power, accuracy, and logical keenness seems to differ widely in the different schools. A disappointingly large number seem to put no special stress on accuracy.

Owing to the recent establishment of this type of school in this country the curriculum is yet in a formative state. Undoubtedly a satisfactory course for these schools will be developed, probably as a part of the larger problem of industrial education. On account of its practical character and wide application, mathematics is sure to occupy an important place in this kind of education. That the subject matter, problems, and applications of mathematics to various industries can and should be much better presented than has been done in the past seems obvious.

SUPPLEMENTARY REPORT ON THE INDUSTRIAL SCHOOL OF SECONDARY AND INTERMEDIATE GRADE.

Within the last decade the industrial school of secondary and intermediate grade has assumed an important place in our system of education.

It has been felt that both elementary and secondary schools have made it too generally their aim to prepare the pupil for eventual higher education and have in consequence laid too little stress on preparing him for immediate vocational efficiency. To this may be attributed the sudden decrease in school registration at the end of the period of compulsory attendance.[1]

To satisfy this demand for vocational education of intermediate and secondary grade numerous technical and industrial schools have been established and are being established in all parts of the country.

In order to determine the nature of the mathematical curricula of these schools, and in particular to determine what mathematics their authorities consider essential for industrial efficiency, the following circular was sent to about 40 schools listed in Bulletin II of the National Society for the Promotion of Industrial Education:

I. What is the mathematical preparation required of a candidate for admission to your school?

---

[1] Massachusetts Commission on Industrial and Technical Education. Report, 1906.

II. What textbook, if any, do you use in each branch of mathematics taught in your school?

III. Is the mathematics in your school similar in scope, content, and method to that in the ordinary academic secondary school?

IV. If it is not, is it practical, applied, or shop mathematics?

V. What topics, parts, or subdivisions of algebra, geometry, arithmetic, etc., do you include in your course in mathematics?

VI. How much time (hours a week, weeks a year, and years) do you devote to each branch of mathematics?

VII. Can you furnish me with a few specimen problems used by you in the various branches of your practical mathematics?

VIII. Could you send me specimen examination papers which you have set for your classes in the various branches of mathematics?

Replies to these questions were received from 21 of the 40 schools. It appears from the answers that of these 21 schools 2 are of college grade, and hence need not be considered here; 6 are high schools of the usual type, listed as industrial schools because they offer a certain amount of shopwork; 2 are schools for apprentices; and the remaining 11 are really industrial schools.

The replies may be summarized in part as follows:

I.—*Preparation for entrance.*

	Number of schools.
No preparation required for entrance	1
Knowledge of the fundamental operations	2
Mathematics of the first six grades of the elementary school	3
Mathematics of the first eight grades	13

II.—*Use of texts.*

It appears that a number of the schools use no text in certain or in all of the subjects, but that when texts are used they are of the type employed in general secondary schools. The results in detail are as follows:

Subjects.	Number of schools using—		
	No text.	Text.	Total.
Elementary algebra	8	11	19
Plane geometry	9	10	19
Solid geometry	1	4	5
Plane trigonometry	5	5	10
Advanced algebra	....	4	4
Elements of bookkeeping	....	2	2

III.—*Scope and content of course.*

Most of the schools aim at including in their course the mathematical work ordinarily given in the general secondary school. They ordinarily give in addition instruction in such matters as percentage, mensuration, and the use of formulæ.

All claim a scope and content of course more extensive than that of the general secondary school.

IV. *Is the work practical, applied, or shop mathematics?*

The schools all claim that it is eminently so.

VII. *Problems.*

There is a marked difference in the character of the problems submitted by schools which are merely general high schools with shops attached and those which are essentially industrial schools. In schools of the former class the problems are for the most part similar to those found in the mathematical tests used in general secondary schools. In a long list of problems submitted by a school of this class, the nearest approach to an industrial problem is the following: "In a hexagonal nut the distance across the flats is $1\frac{1}{2}$ times the diameter of the bolt plus $\frac{1}{8}$ inch. The thickness of the head is one-half the distance across the flats. A hexagonal nut is $\frac{3}{4}$ of an inch on a side. Find the distance across the flats, the approximate diameter of the bolt, and the thickness of the head." The problems submitted by schools of the latter class are apparently drawn from industrial practice. They involve, however, no mathematics beyond mensuration and the arithmetic of the elementary schools.

If one is to judge the mathematical needs of the secondary industrial schools on the basis of the problems submitted in answer to the questionnaire, it seems that direct use is made of little mathematics beyond arithmetic and certain elementary facts of algebra, geometry, and trigonometry. Nevertheless, these schools apparently cover the traditional mathematical curriculum of the secondary school. It would appear that in the present early stage of their evolution these schools have not yet satisfactorily adjusted the content of their courses to the concrete external demands of the shop, on the one hand, and the more abstract internal requirements of good pedagogy, on the other. There are certain problems frequently arising in machine-shop practice which do not appear among the problems submitted, and which require for thorough comprehension much more than the usual mathematical course of the secondary school. The operation of "compound indexing" on the milling machine requires the solution of a linear indeterminate equation. Certain operations with the screw-cutting lathe require the approximate representation of a given rational or irrational number by the convergents of a continued fraction and the determination of the error of the approximation. The fact that such convergents give the best approximation obtainable by means of rational fractions of denominator not exceeding that of the convergent is of prime importance here. Not only this, but the following problem often arises: To express with as great approximation as possible a given irrational or rational number by means of a product, $\frac{n_1}{m_1} \cdot \frac{n_2}{m_2}$ of rational fractions in which n_1, n_2, m_1, m_2 are comparatively small integers.[1]

In certain industrial schools mathematics, though definitely included in the course of study, is not taught as a separate and distinct subject. It is simply introduced as the student happens to strike some phase of work in the shops or drafting room which requires the knowledge of a certain fact. The fact is brought out for his immediate use. In this way he obtains his mathematics.

The question now arises, How much of this instruction is valuable? How long can the student, so taught, retain the knowledge of the facts given him? Can he, moreover, appreciate the facts brought to his attention?

In the first place, such instruction can hardly develop any originality on the part of the pupil. In the second place, he has not the apperceptive mass, from which the various mathematical facts and relations can be drawn out. At best, he can only be made to see that the statements made to him are plausible. At most, he sees only a glimmer of light, and then comes total darkness. When one realizes how difficult to most pupils are certain propositions in geometry, what must one's judgment be upon a method of teaching which tries to impress merely isolated mathematical facts upon the mind—upon a mind, moreover, which has not been prepared by constant drill to recognize intuitively mathematical relations. The conclusion must be that such teaching can not give the student power or lasting knowledge. Often no better teaching can be done on account of the lack of time, but such teaching should not be designated as instruction in mathematics.

In conclusion it may be said that in schools in which mathematics is taught as a distinct subject, the course has not yet been developed which will provide thorough and adequate preparation to attack with full understanding the problem of the shop. It is doubtful whether such preparation can be supplied with much less attention to the formal and logical element than is now customarily given.

The instruction in schools which have no separate mathematical course must, in its present condition, at best be regarded as an undesirable though possibly necessary concession to the demands of a brief course.

The problem of constructing a course which shall be both mathematically and industrially satisfactory is as yet unsolved, largely for the reason that the schools of both types are as yet only in an experimental state.

---

[1] "A Practical Treatise on Gearing," Brown & Sharpe Manufacturing Co., Providence, R. I.